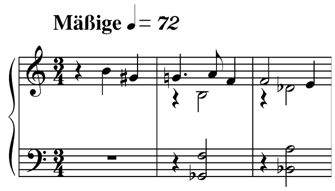
Consider (and play) the opening to Schoenberg's *Three Piano Pieces*, Op. 11, no. 1 (1909):



If we wish to understand how it is organized, we could begin by looking at the melody, which seems to naturally break into two three-note cells: , and . We can see right away that the two cells are similar in contour, but not identical; the first descends **m**3rd - m2nd, but the second descends **M**3rd - m2nd.

We can use the same method to compare the chords that accompany the melody. They too are similar (both span a M7th in the L. H.), but not exactly the same (the "alto" (lower voice in the R. H.) only moves up a diminished 3rd (=M2nd *enh.*) from B to Db, while the L. H. moves up a M3rd).

Let's use a different method of analysis to examine the same excerpt, called SET THEORY.

WHAT?

• SET THEORY is a method of musical analysis in which PITCH CLASSES are represented by numbers, and any *grouping* of these pitch classes is called a SET.

• A PITCH CLASS (**pc**) is the *class* (or *set*) of pitches with the same letter (or solfège) name that are octave duplications of one another. "Middle C" is a *pitch*, but "C" is a *pitch class* that includes middle C, and all other octave duplications of C.

WHY?

Atonal music is often organized in pitch groups that form cells (both horizontal and vertical), many of which relate to one another. Set Theory provides a shorthand method to label these cells, just like Roman numerals and inversion figures do (i.e., ii $^{6}_{4}$) in tonal music. We use numbers in musical analysis for scale degrees, chord inversions, added notes, and voice-leading (i.e., $^{6-5}_{4-3}$), and this is just one more musical context in which they can be used.

☞ There are different conventions in place for identifying notes. You are probably accustomed to using letter names, but in many countries, solfège is used. There are discrepancies within both of these systems between some of the different countries that use them. In Germany, for example, "H" refers to Bb, and in most European countries, "Si" is used instead of "Ti," whereas in other countries "Si" refers to a sharpened "So." Indian classical music has a different note-labeling system, as do other world regions. There is no single, "standard" note-labeling system, and the use of integers is just one more way of doing so that also happens to be both logical and consistent.

HOW DOES IT WORK?

(i) Identify a melodic or harmonic cell in a composition, such as: , or .

(ii) Rewrite the notes in ascending order, and so that they fit within one octave (if necessary), then number each pitch in the cell, making the lowest one = 0, increasing by 1 for each semitone.

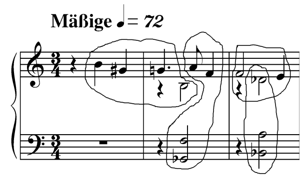
Thus,  becomes 

, and  becomes 

.

(iii) The **label** for the cell (called a "set") is created by arranging the numbered pitches in **ascending order** and in the **most compact form possible**, assigning 0 to the first note. *By "most compact form" we mean the form with the smallest interval between the first and last notes.* For the opening three pitches of the Schoenberg example above this would be [0, 1, 4].

☞ Label the circled sets in the first two bars of the Schoenberg example below as either [0, 1, 4] or its inversion, [0, 3, 4]:



☞ Now continue circling and labeling each occurrence of these two sets in the remaining bars below:

WHY ARE WE OBSSESSING OVER THE [0, 1, 4] AND [0, 3, 4] SETS? WHAT ABOUT THE *NEXT* 3 MELODY NOTES?

Good question! We began by suggesting that the melody in the 1st 3 bars seems to naturally break into two cells,  and , yet we have spent all our time looking for examples of the 1st set and its inversion, and *no* time looking for examples of the 2nd set and *its* inversion. **It is important not to overlook other set possibilities beyond the one or two that you may start with**. The only way to determine if the 2nd 3 melody notes form a set that is of structural significance is to look for other examples of that set [0, 1, 5]. Do so now. Please. ☺

• **Another potential problem is finding the set you are looking for in places that make little musical sense**. The top voice has a G# in m. 1, G in m.2, and E at the end of m.3, which form a [0, 3, 4], but does it make musical sense to call those three notes a set?

NORMAL ORDER:

The arrangement of a set of pitch classes in **ascending order** and **most compact form** is called the set's "**normal**" order (or normal form), and is the equivalent of deriving a root-position chord or scale from the notes used in a section of a tonal work; both result in a label that tell us something about the organization of a group of pitches.

For example, we might say of a tonal composition that it begins in Bb major, then modulates to G minor, and we could similarly comment that the opening three circled pitch collections above use the [0, 1, 4] set, while the last one uses [0, 3, 4] (in bar 3). "Bb major" and "G minor" are labels, as are [0, 1, 4] and [0, 3, 4].

☞ To arrive at the **most compact form**, you could write out the set in *every* possible rotation (like writing a chord in every possible inversion), then pick the one with the smallest interval between the first and last pitch (but see top of next page for what to do when there is more than one set rotation with equally-compact forms).

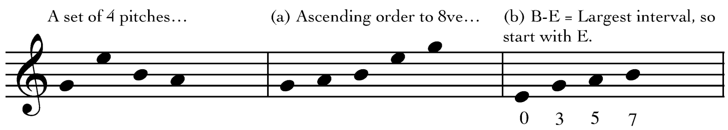
☞ However, there is a faster method (see box below)…

A *FASTER* METHOD TO DETERMINE NORMAL ORDER:

(a) Write *any* form of the set in **ascending** order **within an octave**, but repeat the first note an 8^{ve} higher at the end.

(b) Next, identify the **adjacent** pair of notes that forms the largest interval and re-write the set so that the **second** of those two notes is the starting, or lowest, note.

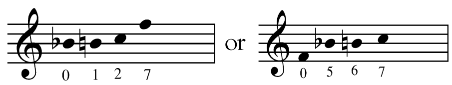
- This produces the set rotation with **the smallest possible interval between the first and last notes**.



DRILL — Write the **normal order** for each of the following pitch collections:

1) E D B G 2) F C B E 3) F Eb A Ab D 4) B F Bb C 5) C# F# A# D# G

- You may have noticed that there are *two* occurrences of the largest interval in #4, resulting in:



- A similar "tie" occurs in #5.

☞ **IN CASE OF A TIE: If there are two or more possible candidates for a set's normal order** (two or more equally-compact set rotations with an identical interval between the first and last notes), **compare the interval between the first and *second-last* note in each set**; the set with the smaller of these is the *correct* normal order. If this results in another tie, then compare the interval between the first and *third-last* note in each set, and, if necessary, continue the process until the tie is broken. If the tie is never broken, then either can be the correct normal order.

NB: (i) ENHARMONICALLY EQUIVALENCY: Enharmonic spellings may be used for convenience and make no difference.

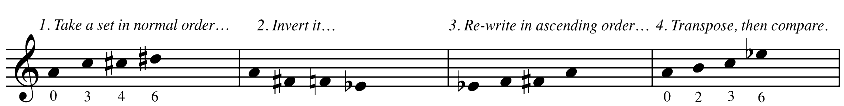
(ii) OCTAVE (or TRANSPOSITIONAL) EQUIVALENCY: The octave or transposition in which pitches are notated does not matter, because we are dealing with *pitch classes*.

(iii) INVERSIONAL EQUIVALENCY: **A set and its inversion (mirror) are two representations of the same thing**, much like a fugue subject and its inversion, and are therefore considered as belonging to the same *SET CLASS*.

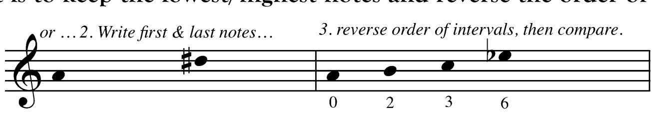
(iv) Due to inversional equivalency, **any normal form of a set must be compared with its inversion to determine which is the "better" of the two** (using the tie-breaking method described above); this is called the set's "**BEST** NORMAL ORDER."

- *The <u>best</u> normal order usually has the smallest intervals on the left.*

☞ **HOW TO INVERT PC SETS:** You can invert a set the traditional way (e.g.: a m3^{rd} up inverted becomes a m3^{rd} down), then rewrite the pitches in ascending order, then transpose the inversion so that it begins on the same pitch as the original form, and finally compare the set with its inversion in order to find the <u>best</u> normal order:



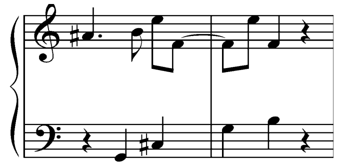
☞ **A faster way to invert a set is to keep the lowest/highest notes and reverse the order of the intervals between them:**



☞ **Another fast inversion method is to use numbers only**: (i) Keep **1st** and **last** number (i.e., **0**, **6**), then (ii) subtract the numbers in the middle from the last number (i.e., 6-3=**3**; 6-4=**2**), and (iii) reverse them (i.e., [**0, 2, 3, 6**]).

DRILL — The following are sets from the 2nd of Schoenberg's op. 11 piano pieces. Are they in **best** normal order? Compare each set with its inversion to find out (for additional practice, do this drill with previous sets in this handout):

If a set results in the same "best" normal order in two different transpositions it is called **transpositionally-symmetrical**. Assume all pitches in the next excerpt form a single set, then write the pitches in best normal order in two different but equivalent transpositions:



PRIME FORM, SET CLASS, FORTE NAME, AND OTHER TERMINOLOGY

• The **Best Normal Order** is usually written using letter names for pitch classes (e.g, C D Eb F#), or music notation.

• The numerical **label** for the best normal order**,** where the lowest pitch = 0, is called the **Prime Form**. The prime form for [0 3 4 6] is [0 2 3 6]. Can you explain why?

• A **Set Class** includes *all* sets that are related by transposition and/or inversion. [B, C, Eb], [C, C#, E], [C#, D, F], etc., all belong to the same set class as [B, D, Eb], [C, Eb, E], [C#, E, F], etc.

• The **label** for a set class is usually its **prime form** (e.g., [0 2 3 6]), or its **Forte name** (4-12).

• A **Forte name** is the label given to *every* possible prime form by theorist Allen Forte based on its cardinality and relative position within its cardinality. See Appendix C in your text for a list of these.

• **Cardinality** is the number of pitch classes in a given musical set (or the number of elements in a mathematical set).

• Two-pitch sets are called **dyads**, and three-pitch sets are **trichords**. Sets of higher cardinalities are called **tetrachords**, **pentachords**, **hexachords**, **heptachords**, **octachords**, **nonachords**, **decachords**, **undecachords**, and **dodecachords**. I include the last few terms mainly for your amusement; you may wish to astound and entertain others by working them into your conversation at some point. Most sets are between 2-6 notes.

• Pitch classes may be referred to by letter name, solfège name, or number. If using numbers, the default is to let C = 0, C# = 1, D = 2, etc. However, as explained above, we usually let the **lowest** pitch in a set = 0, then number the remaining pitches accordingly, which is similar to the concept in solfège of movable Do (versus fixed "Do").

• A **subset** is a set that is part of a larger set; a larger set may include several overlapping or distinct subsets.

• An **aggregate** is a collection of 12 different pitches.

• The **complement** of a set is the collection of pitch classes that are not members of the set.

HOW DOES VECTOR ANALYSIS (INTERVAL VECTOR) WORK?

1) Create 6 "columns" or "slots", enclosed by angle brackets. Each slot represents a different interval "class." *Octave displacements are disregarded in interval classes, so* ***any interval and its inversion are of the SAME class*** (so a P. 5^{th} is the same class as P. 4^{th}). Thus, there are only **6 possible interval classes** (there are really 7 if you count the unison/octave, but unisons and octaves are never part of pc sets because each pc is represented only once):

< _	_	_	_	_	_ >
m2/ M7	M2/ m7	m3/ M6	M3/ m6	P4/ P5	+4/ °5

2) Now count all possible intervallic relationships contained in a particular set, listing your totals in each column (the interval vector will be the same irrespective of the order of pitches within the set; do you know why?). Start by listing the interval between the 1^{st} note and the 2^{nd}, then from the 1^{st} to the 3^{rd}, and so on. Continue by listing the interval from the 2^{nd} note to the 3^{rd}, and from the 2^{nd} note to any subsequent notes; than include the intervals from the 3^{rd} note to any subsequent notes, and so on until you run out of notes. Practice this now with the following sets:

DRILL – Calculate the interval vectors for the following sets, as well as for previous sets for additional practice:

[F, F#, G#, C] [E, G, Ab, Bb] [0,3,4,6] [0, 1, 4, 5]

WHAT IS THE POINT OF VECTOR ANALYSIS?

☞ **It gives us additional information on a set's harmonic content**, much like the label "C major chord" tells us something about both pitches and interval content in that chord. This is sometimes useful, but, as our text points out, there are cases where two different set prime forms have some the same vector analysis, like:

[G, Ab, Bb, D] and [G, Ab, B, C#] (do a vector analysis of both now)

☞ Two sets that share the same interval-class vector are said to be "Z-related."

- Do a vector analysis of a root -position C major chord. Then do vector analyses of 1st-inversion and 2nd-inversion C major chords. Try other major chords. What conclusion do you draw from this exercise?
- Do a vector analysis of a C minor chord, and compare with that of the C major chord. Conclusions?
- Compare the prime form of a C major chord with that of a C minor chord. Conclusions?

TO WHOM DO WE OWE THIS PLEASURE (/PAIN/BEWILDERMENT, ETC.)?

The American composer Howard Hanson is credited (in the "Set Theory" article in Wikepedia) as having developed most of the ideas pertaining to the application of set theory to music in *Harmonic Materials of Modern Music: Resources of the Tempered Scale* (1960), but another American composer, Milton Babbitt, wrote a 1946 study called 'The Function of Set Structure in the Twelve-Tone System', which would suggest he was investigating the application of mathematical sets to music considerably earlier. Other pioneers in the field include Allen Forte (*The Structure of Atonal Music* - 1973), George Perle (*Serial Composition and Atonality* -1962), and David Lewin (numerous articles).